

Description for rotating C_{60} fullerenes via Gödel-type metric

Everton Cavalcante*

*Departamento de Física, Universidade Federal da Paraíba,
Caixa Postal 5008, 58051-970, João Pessoa, PB, Brazil;
Centro de Ciências Exatas e Sociais Aplicadas,
Universidade Estadual da Paraíba, Patos, PB, Brazil*

Josevi Carvalho†

*Unidade Acadêmica de Tecnologia de Alimentos,
Centro de Ciências e Tecnologia Agroalimentar,
Universidade Federal de Campina Grande,
Pereiros, Pombal, PB 58840-000, Brazil*

Claudio Furtado‡

*Departamento de Física, Universidade Federal da Paraíba,
Caixa Postal 5008, 58051-970, João Pessoa, PB, Brazil*

Abstract

In this contribution a geometric approach to describe a rotating fullerene molecule with I_h symmetry is developed. We analyze the quantum dynamics of quasiparticles in continuum limit considering a description of fullerene in a spherical solution of the Gödel-type space-time with a topological defect. As a result, we study the molecule in a rotating frame. Also we combine the well know non-Abelian monopole approach with this geometric description, including the case of the presence of the external Aharonov-Bohm flux. The energy levels and the persistent current for this study are obtained, and we show that they depend on the geometrical and topological properties of the fullerene. Also, we verify recovering of the well known results for limiting cases.

PACS numbers: 03.65.Pm, 03.65.Ge, 78.30.Na, 73.23.Ra

Keywords: fullerene, geometric theory of defect, Dirac equation in curved space

* Electronic address: everton@ccea.uepb.edu.br

† Electronic address: josevi@ccta.ufcg.edu.br

‡ Electronic address: furtado@fisica.ufpb.br

I. INTRODUCTION

The fullerene discovered by Kroto, Curl and Smalley [1] in 1996 is a new allotropic form of carbon with spherical symmetry. This molecule is formed by arranging 12 pentagonal rings combined with 20 hexagonal rings of carbon atoms. Thus, the fullerenes are carbon allotropic forms which can be described by curved space, that is, the sphere. The geometric description of the fullerene was firstly developed by Gonzales, Guinea and Vozmediano [2, 3] with use of a tight-binding model. They showed that the Fermi surface is reduced to two points, \mathbf{K} -points, located in the Brillouin zone, and the electrons in this structure obey the massless Dirac equation for fermions [4–6]. The fullerene can be described by a spherical lattice of Ih symmetry, where the presence of pentagonal rings demonstrates the impact of topological defects in this spherical structure. These defects are responsible for the presence of pentagons and are denominated as disclination. The formation of this disclination in a crystalline structure can be viewed with the called Volterra process [7]. In this process, disclination can be created by the cut and glue process. Disclinations are topological defects associated with curvature and are characterized by the Frank vector. A geometric formulation for this topological defects was developed by Katanaev and Volovich [8], in this theory, the continuum description for disclinations and dislocation is given with use of curved space with curvature and torsion.

In two-dimensional systems of condensed matter the presence of disclinations is quite relevant for physical properties of this systems. The study of the influence of disclinations in curved structures of graphene employing Dirac equation approach to describe the dynamics of a quasiparticle in this material, have been relatively well performed in the Refs. [9–18]. The effect of curvature introduced by disclination in this carbon structure induces an effective gauge field generated by the variation of the local reference frame [4–6, 9, 17–19]:

$$\oint \omega_\mu dx^\mu = -\frac{\pi}{6}\sigma^3, \quad (1)$$

forcing a mixture of the Fermi points (\mathbf{K}_\pm), and generating a non-Abelian gauge field (or a K-spin flux) as demonstrated by [17, 18], which compensates the discontinuity of the Bravais labels (A/B):

$$\oint A_\mu dx^\mu = \frac{\pi}{2}\tau^2 \quad (2)$$

where τ^2 is the second Pauli matrix that mixes the \mathbf{K}_+ and \mathbf{K}_- components of the spinor on

the \mathbf{K}_{\pm} space. In other words, the mixture of Fermi points induces an effective field arising due to a fictitious magnetic monopole in the center of the Ih fullerene by replacing the fields of 12 disclinations.

The first model to describe electrons in C_{60} was proposed by Gonzales, Guinea and Vozmediano [2, 3]. In this model the fullerene is described by a non-Abelian magnetic monopole introduced in the centre of a two-dimensional sphere. The field produced by this monopole represents the fields of twelve disclinations presenting in the Ih symmetry of the fullerene molecule. This model establishes the first approach to study of fullerene using a Dirac equation to describe low energy electrons in C_{60} molecule. Another description for fullerene in which a gauge field theory is used to describe topological defects, was used by Kolesnikov and Osipov [20]. In this model the pentagonal rings in fullerene are described by two gauge fields, with one of them is used to introduce the elastic properties of disclinations, and the second gauge field, is a non-Abelian gauge field describing the \mathbf{K} -spin fluxes. In recent years, some studies of fullerene molecules were employing the continuous model to describe electronic properties of spheroidal fullerenes, see [21–23]. Recently, two of us have used the geometric theory of defects to describe fullerene molecule. In this study the well known non-Abelian monopole approach [2, 3] was combined with the geometric theory of defects [8] to obtain the energy levels and the persistent current [24]. Therein, the Dirac equation for a spherical geometry with topological defects and in the presence of an Aharonov-Bohm flux was solved, and the eigenfunctions and eigenvalues for this problem were exactly determined.

Recently, the study of theoretical models considering the influence of rotation in curved carbon structures has been carried out. Shen and He [26] used Schrödinger equation to study the arising of Aharonov-Carmi effect [25] on a rotating molecules of fullerene. In a recent study, Lima et. al. [27, 28] investigated the effect of rotation in the electronic spectrum fullerene molecules using non-Abelian gauge fields [2, 20] to describe the defect and non-inertial effects, including rotation of the reference frame in the Dirac equation. The influence of rotation in nanotubes of carbon was also investigated in [29]. In the present contribution, we investigate the influence of rotation in the spectrum of energy fullerene molecule using a geometric description and the well known non-Abelian monopole approach [2, 3]. The geometry of spherical rotating body is introduced via a three-dimensional spherical Gödel solution with presence of topological defects.

Here, we associate a doublet of spinors interacting with a characteristic curvature of space

and with a curvature accumulated in a pentagonal defects (conical singularities) via gauge field (A_μ) from the fictitious magnetic monopole in the centre. In order to put the contents of spherical fullerene under rotation, we assume the mapping of a Ih fullerene in a spherical three-dimensional solution ($l^2 < 0$) of a Gödel-type space-time with $z = 0$ (See Appendix). Also, we study the change in spectrum of the C_{60} molecule when this is crossed by a magnetic flux tube in the direction z .

This paper is organized as follows. In Section 2, we give a brief review of the geometric approach for a fullerene molecule, where its dynamics is well represented by an effective Dirac equation in curved spaces. We show how to put this content of matter under rotation through a model of continuous in the spherical Gödel-type metric. In Section 3, we consider the presence of a Aharonov-Bohm magnetic flux and discuss how the presence of magnetic flux shifts the spectrum. Also, we show the emergence of the persistent current, and we compare how the non-inertial characteristics change the spectrum and this persistent current. In Section 4, we discuss the results obtained in the paper.

II. GEOMETRIC APPROACH TO A FULLERENE MOLECULE IN A NON-INERTIAL FRAME

In this section we present the approach used to describe a fullerene molecule in a non-inertial frame. For this we consider the mapping of a fullerene under rotation in a space-time with spherical symmetry descendant of a Gödel-type solution of the Einstein field equations. In this space-time, the conical singularities **induce** a non-zero curvature in space. So, we combine two treatments: a non-Abelian monopole approach that is already very well known [44, 45] and the geometry theory of defects of Katanaev and Volovich [8], based on the equivalence between three-dimensional gravity with torsion and the theory of defects in solids. Thus, the molecule with conical singularities can be described by a Riemann-Cartan geometry. The choice of one spherical Gödel-type solution is motivated by the fact that it represents a cosmological solution where the content material is rotated. It is interesting to note that the relationship between the geometric approach and the physics of the electronic structure of the molecule is well described at low energies, around a few tens of eV around the **K**-points. It is still useful for elucidating a long-distance physics, since we note that the eigenfunctions of the low-energy levels do not oscillate too rapidly at a distance [63]. Thus,

we consider the situation in the neighbourhood of the defects. In this geometric approach, we consider the fullerene molecule under rotation in terms of a two-dimensional spherical geometry with a rotation is labelled as Ω . Now, we consider Gödel solution of spherical symmetry, that correspond a rotating body with vorticity (rotation) about the z -axis (see Appendix for details of Gödel-type metric solution in Gravitation).

In this solution which was well studied in the context of gravity by [35–37], the Gödel metric is described by following line element:

$$ds^2 = - \left[dt + \alpha \Omega \frac{\sinh^2(lr)}{l^2} d\phi \right]^2 + \alpha^2 \frac{\sinh^2(2lr)}{4l^2} d\phi^2 + dr^2 + dz^2, \quad (3)$$

with the variables (r, ϕ, z, t) can take, respectively, the following values: $0 \leq r < \infty$, $0 \leq \phi \leq 2\pi$, $-\infty < (z, t) < \infty$, and α is related to the angular sector λ removed/inserted from/into a spherical sheet in order to form the two conical defects in the sphere by the expression $\alpha = 1 \pm \lambda/2\pi$. Indeed, to respect the symmetries of the carbon network, λ can only be $\pm N\pi/3$, where N is an integer in the interval $(0, 6)$. Values of α in the interval, $0 < \alpha < 1$, mean that we remove a sector of the sphere to form two topological defects in the antipodal point. Taking into account the theory of Katanaev and Volovich [8], the elastic continuous medium with topological defect is represented by a three-dimensional space with curvature and torsion. We can see that the spherical Gödel-type solution as a description of a spherical body with defects rotating around the z -axis. As we are studying the fullerene like a C_{60} buckyball, it is appropriate to do $dz = 0$ in Eq. (3), and we introduce the new convenient coordinates: $R = i/2l$ and $\theta = r/R$, resulting in

$$ds^2 = - \left[dt + 4\alpha\Omega R^2 \sin^2 \left(\frac{\theta}{2} \right) d\phi \right]^2 + R^2 (d\theta^2 + \alpha^2 \sin^2 \theta d\phi^2) \quad (4)$$

It is noteworthy that we are adopting natural units $c = \hbar = G = 1$. Also, when we consider $\Omega = 0$ and $\alpha = 1$, the metric reduces to the Minkowski space. When $l^2 = \Omega^2/2$ and $\alpha = 1$ we recover the original solution obtained by Gödel [32]. This metric corresponds to the elastic space-time surrounding the defect and provides all the informations required to characterize the physical system.

The bases of this space-time are known as tetrads $(e^a_\mu(x))$, which are defined at each point in space-time by a local reference frame $g_{\mu\nu}(x) = \eta_{ab} e^a_\mu e^b_\nu$. The tetrad and its inverse, $(e^\mu_a = \eta^{ab} g^{\mu\nu} e^b_\nu)$, satisfy the orthogonal relationships: $e^\mu_a e^b_\mu = \eta^{ab}$, $e^\mu_a e^\mu_b = \delta^a_b$, $e^\mu_a e^\mu_\nu = \delta^\mu_\nu$, and

map the space-time reference frame via the local reference frame [64]:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = e_\mu^a e_\nu^b \eta_{ab} dx^\mu dx^\nu = \eta_{ab} \theta^a \theta^b. \quad (5)$$

Greek indices (μ, ν) run for space-time frame coordinates and the Latin indices (a, b) run for local frame coordinates. For the local reference frame $(\theta^a = e_\mu^a(x) dx^\mu)$, we choose the tetrads:

$$e^a_\mu = \begin{pmatrix} e^0_t & e^0_\theta & e^0_\phi \\ e^1_t & e^1_\theta & e^1_\phi \\ e^2_t & e^2_\theta & e^2_\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 & 4\alpha\Omega R^2 \sin^2(\frac{\theta}{2}) \\ 0 & R & 0 \\ 0 & 0 & \alpha R \sin \theta \end{pmatrix}. \quad (6)$$

The one-form or spin connections $(\omega_\mu^a{}_b)$ are obtained by the variation of the local reference frame along the a closed curve, δe_μ^a . Namely

$$\omega_\mu^a{}_b = -e_\beta^a (\partial_\mu e_b^\beta + \Gamma_{\mu\nu}^\beta e_b^\nu), \quad (7)$$

where $\Gamma_{\mu\nu}^\beta$ are the Christoffel symbols. Another most immediate way to obtain the one-form connection is through the first of the Maurer-Cartan structure equations:

$$d\theta^a + \omega^a{}_b \wedge \theta^b = 0. \quad (8)$$

For the one-forms we get the following connections $(\omega_\mu^a{}_b = \omega_\mu^a{}_b dx^\mu)$: $\omega_\phi^0{}_1 = -\omega_\phi^1{}_0 = 2\alpha\Omega R \sin \theta$, $\omega_\phi^2{}_1 = -\omega_\phi^1{}_2 = \alpha \cos \theta$, and $\omega_\theta^0{}_2 = -\omega_\theta^2{}_0 = 2\Omega R$. Thus the spinorial connections $(\Gamma_\mu(x) = \frac{i}{4}\omega_{\mu ab}\Sigma^{ab})$, are described as the components of the doublet related to the **K**-points as a matrix in $(2+1)$ -dimensions where the Dirac matrices γ^a are reduced in our case to the Pauli matrices $\gamma^a = \sigma^a$, and the matrix $\sigma^0 = I$ is the 2×2 identity matrix, thereby determining the pseudo-spin degrees of freedom. So, the resulting spinorial connections read as follows:

$$\begin{cases} \Gamma_\phi = \frac{i}{2}(\alpha \cos \theta \sigma_3 - 2\alpha\Omega R \sin \theta \sigma_2) \\ \Gamma_\theta = i\Omega R \sigma_1 \end{cases} \quad (9)$$

Next, we solve the Dirac equation on the surface of a sphere with a fictitious magnetic monopole at its centre. The charge g of the fictitious magnetic monopole is adjusted by adding up the individual fluxes of all the lines[3]

$$g = \frac{1}{4\pi} \sum_{i=1}^N \frac{\pi}{2} = \frac{N}{8}, \quad (10)$$

where N being the number of conical singularities on the surface. Note that for the buckyball C_{60} , the structure is that of a truncated icosahedron (where $N = 12$). Thus we have $g = \frac{3}{2}$, which is compatible with the standard quantization condition of the monopole charge [45, 65]. The spectrum is obtained by solving the covariant Dirac operator,

$$-i\hbar V_f \sigma^a e_a^\mu (\nabla_\mu - iA_\mu) \psi = 0, \quad a = 0, 1, 2, \quad \mu = t, \theta, \phi, \quad (11)$$

knowing that $\nabla_\mu = \partial_\mu - \Gamma_\mu$, and now we assume $V_f = 1$ is the Fermi velocity. In such way there is a non-Abelian gauge field (A_μ) that arises due to the \mathbf{K} -spin fluxes. This 't Hooft-Polyakov monopole must be compatible with the standard quantization condition, and it is well reported by

$$A_\phi = g \cos \theta \tau^{(2)} = \frac{3}{2} \cos \theta \tau^{(2)}. \quad (12)$$

Furthermore, it is noteworthy that $\tau^{(2)}$ acts only in the space of \mathbf{K}_\pm spinor components, while σ^a , existing in spinorial connection (Γ_μ), only acts on the geometry. When these matrices operate in different subspaces, we can decouple the doublet (ψ^\pm). In this case we will have a rotation in the monopole field, thus we obtain a frame where $\tau^{(2)}$ is diagonal:

$$\oint A_\mu^{Rot.} dx^\mu = A_\phi^{Rot.} = U^\dagger A_\phi U = A_\phi^k = \begin{cases} g \cos \theta, & \text{if } k = (+) \\ -g \cos \theta, & \text{if } k = (-) \end{cases} \quad (13)$$

where

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \quad (14)$$

Based on a rotation, we can separate the \mathbf{K}_\pm pseudo-spin components on the doublet. In this case (11) is reduced to the following

$$-i\hbar \sigma^a e_a^\mu (\partial_\mu - \Gamma_\mu - iA_\mu^k) \psi^k = 0 \quad (15)$$

Hence we can carry out our study in the context of eigenvalues problem just defining explicitly the quantum numbers for ∂_t and ∂_ϕ . That is, we employ an *ansatz*: $\psi^k(t, \theta, \phi) = \exp(-i\epsilon t) \exp(im\phi) \psi_{n,m}^k(\theta)$. However, before that, let us consider the influence of the Aharonov-Bohm magnetic flux (Φ_B) generated by a magnetic string passing through the north pole to the south pole of sphere.

III. THE ROTATING FULLERENE IN THE PRESENCE OF AHARONOV-BOHM FLUX TUBE

In this section, we consider the geometric model described by the Dirac equation (15) in the presence of Aharonov-Bohm flux. This flux represents a magnetic string that goes from pole to pole in the C_{60} buckyball. This is the same as assuming the molecule to be under the influence of a magnetic field $\vec{B} = B_z \hat{z}$ where $B_z = \Phi_B \delta(r)$. This flux is associated with a vector potential in the local reference frame by [24, 66, 67]

$$A_{\phi,MS} = \frac{\Phi_B}{2\pi}. \quad (16)$$

Thus, using the minimal coupling to include the potential vector (16) in Dirac equation (15), after replacing the inverse of the tetrads (e_a^μ) and spinorial connections ($\Gamma_\mu(x)$), and considering $\lambda = \frac{eR}{\hbar}$, we find

$$\left[\frac{d}{d\theta} + \left(\frac{1}{2} + k \frac{g}{\alpha} \right) \cot \theta - \frac{k}{\alpha \sin \theta} \left(m - \frac{\Phi_B}{2\pi} + 4\alpha\Omega\lambda \sin^2 \left(\frac{\theta}{2} \right) \right) \right] \psi_{n,m}^{(k)} = i\lambda \psi_{n,m}^{(-k)} \quad (17)$$

Note that we define explicitly the ϕ -coordinate quantum number ($m = j \pm \frac{1}{2} = 0, \pm \frac{1}{2}, \pm \frac{3}{2}, \dots$) and the stationary character of the problem (energy spectrum (λ)) through the *ansatz*: $\psi^k(t, \theta, \phi) = \exp(-i\epsilon t) \exp(im\phi) \psi_{n,m}^k(\theta)$. Also, when $\Omega = 0$, we recover the inertial case described in references [24, 63, 68, 69] in their respective conditions. These two components of the doublet ($\psi_{n,m}^\pm$) combine to give:

$$\left\{ \frac{1}{\sin \theta} \frac{d}{d\theta} \sin \theta \frac{d}{d\theta} - \frac{1}{\sin^2 \theta} \left[\frac{1}{\alpha^2} \left(m - \frac{\Phi_B}{2\pi} \right)^2 - \left(k + \frac{2g}{\alpha} \right) \left(m - \frac{\Phi_B}{2\pi} \right) \frac{\cos \theta}{\alpha} + \frac{g}{\alpha} \left(\frac{g}{\alpha} + k \right) + \frac{1}{4} \right] + \lambda^2 - \frac{1}{4} + \frac{g^2}{\alpha^2} - 4\Omega\lambda \frac{\sin^2 \theta/2}{\sin \theta} \left[k + \frac{2}{\alpha} \left(m - \frac{\Phi_B}{2\pi} - g \cos \theta \right) + 4\Omega\lambda \sin^2 \frac{\theta}{2} \right] \right\} \psi_{n,m}^k = 0 \quad (18)$$

We will follow the same way as in [24]. That is to say that the asymptotic limits do not change when the matter rotates. Moreover, our alternative approach for finding the spectrum $\lambda(\epsilon)$ is based on changing the coordinates $x = \cos^2(\theta/2)$ and the *ansatz*

$$\psi_{n,m}^k = x^{C+} (1-x)^{C-} H_{n,m}^k(x), \quad (19)$$

with

$$C_{\pm} = \frac{1}{2} \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \pm \frac{1}{2} \left(k + \frac{2g}{\alpha} \right) \mp \frac{\Omega}{2\alpha} \left(m - \frac{\Phi_B}{2\pi} + g \right) \right|. \quad (20)$$

Thus, after replacing the *ansatz* (19), we obtain an equation for $H_{n,m}^k(x)$:

$$\begin{aligned} & \left[x(1-x) \frac{d^2}{dx^2} + \left\{ \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) + \frac{1}{2} \left(k + \frac{2g}{\alpha} \right) - \frac{\Omega}{2\alpha} \left(m - \frac{\Phi_B}{2\pi} + g \right) \right| + 1 + \right. \\ & - 2 \left(\left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + 1 \right) x \left. \right\} \frac{d}{dx} - \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| \left(\left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + 1 \right) + \right. \\ & \left. + (1 - 8\Omega^2)\lambda^2 - 2\Omega\lambda \left[1 + \frac{2}{\alpha} \left(m - \frac{\Phi_B}{2\pi} + g \right) \right] - \frac{1}{4} + \frac{g^2}{\alpha^2} \right] H_{n,m}^k(x) = 0. \end{aligned} \quad (21)$$

Note that equation (21) is very similar to the standard hypergeometric equation:

$$\left[x(1-x) \frac{d^2}{dx^2} + \{ \mu + 1 - (\mu + \nu + 2)x \} \frac{d}{dx} + n(n + \mu + \nu + 1) \right] F(A, B, C, x) = 0 \quad (22)$$

Indeed, to ensure the finiteness of the wave function, the (21) should behave as a confluent hypergeometric series. Thus, it is possible to compare (21) to (22), provided that

$$(1 - 8\Omega^2)\lambda^2 - 2\Omega \left[1 + \frac{2}{\alpha} \left(m - \frac{\Phi_B}{2\pi} + g \right) \right] \lambda - \left(n + \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + \frac{1}{2} \right)^2 + \frac{g^2}{\alpha^2} = 0. \quad (23)$$

Thereby, we find the spectrum for the particles in model by solving (23), resulting in

$$\begin{aligned} \epsilon_{n,m} = & \frac{\hbar}{2R(1 - 8\Omega^2)} \left\{ 4\Omega \left[\frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) + \frac{1}{2} + \frac{g}{\alpha} \right] + \right. \\ & \left. \pm 2 \sqrt{2 \left[\frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) + \frac{1}{2} + \frac{g}{\alpha} \right]^2 \Omega^2 - (1 - 8\Omega^2) \left[\frac{g^2}{\alpha^2} - \left(n + \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + \frac{1}{2} \right)^2} \right] \right\} \end{aligned} \quad (24)$$

It is interesting to note that for $\Omega = 0$ we restore the inertial case [24]. Moreover, it is worth noting that the inversely proportional dependence between $\epsilon_{n,m}$ and R do not change the separation between levels. In fact Φ_B operate as a shift in the z -component of the angular momentum. Also it should be noted that when $\alpha = 1$, $\Phi_B = 0$ and $\Omega = 0$, we recover the results obtained by Kolesnikov and Osipov [63]. Also, when $g = 0$ and $\Phi_B = 0$, we replicate the approach stated by Imura [68], for a dynamics of the fermions in a spherical topological insulator. Now, consider the following limit $\Omega \ll 1$ in (24) we obtain the following equation:

$$\epsilon_{n,m} = \frac{\hbar}{2R} \left\{ 4\Omega \left[\frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) + \frac{1}{2} + \frac{g}{\alpha} \right] \pm 2 \sqrt{\left(n + \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + \frac{1}{2} \right)^2 - \frac{g^2}{\alpha^2}} \right\}. \quad (25)$$

Note that the equation (25) is the same found in Ref. [27] for $\Phi = 0$ and $\alpha = 1$. In this way, in the slow rotation limit we recover the results obtained in [27]. Also, the first term is the same contribution obtained by Shen [26] for Aharonov-Carmi [25] effect.

A. The obtaining of the persistent current

Next, we calculate the persistent current for our model of fullerene drilled by a chiral magnetic string. Indeed, persistent currents were first observed in superconducting rings [70], while studying the transition around the critical temperature under the influence of an external magnetic field. Then, after removing the external field, a residual current is observed. It should be noted that these persistent currents are not caused by outside sources, but feature a quantum effect, and present not only in superconductors but also in usual conductor and semiconductor materials. It is calculated using the Byers-Yang relation [71] given by,

$$I = - \sum_{n,m} \frac{\partial \epsilon_{n,m}}{\partial \Phi_B} \Big|_{T=0}. \quad (26)$$

This relation expresses the persistent current for $T = 0$ as a derivative of the energy with respect to the magnetic flux of the chiral string. We now obtain the following expression for the persistent current,

$$I = \frac{\hbar}{2\pi\alpha R} \sum_{n,m} \left\{ 2\Omega + \frac{(1 - 8\Omega^2) \left[n + \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + \frac{1}{2} \right] + 2\Omega^2 \left[\frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) + \frac{1}{2} + \frac{g}{\alpha} \right]}{\sqrt{2 \left[\frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) + \frac{1}{2} + \frac{g}{\alpha} \right]^2 \Omega^2 - (1 - 8\Omega^2) \left[\frac{g^2}{\alpha^2} - \left(n + \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + \frac{1}{2} \right)^2}} \right\}. \quad (27)$$

Note that the persistent current depends on the parameters α and g , which characterize the presence of topological defects, as well as the rotation parameter Ω . We recover the inertial case [24] when $\Omega = 0$. Now, we consider the limit where $\Omega \ll 1$ in (27). In this way we

obtain the following expression for persistent current,

$$I = \frac{\hbar}{2\pi\alpha R} \sum_{n,m} \left\{ 2\Omega + \frac{\left[n + \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + \frac{1}{2} \right]}{\sqrt{\left(n + \left| \frac{1}{\alpha} \left(m - \frac{\Phi_B}{2\pi} \right) \right| + \frac{1}{2} \right)^2 - \frac{g^2}{\alpha^2}}} \right\}. \quad (28)$$

Notice that in the slow rotation limit $\Omega \ll 1$, we can separate two contributions for persistent current. The first term in Eq. (28), directly associated to Ω , and the second term, that is the same found in Ref. [24].

IV. CONCLUSION

We have investigated a geometric description of the rotating C_{60} fullerenes within the continuum field theory approach. For this model we map the C_{60} fullerenes with Ih symmetry in a two-dimensional spinning sphere (about the z-axis) with topological defects, using a Katanaev-Volovich theory of a continuous media [8]. Specifically we describe the matter contents of the molecule like a homogeneous Gödel-type metric with spherical symmetry [32, 35–37].

Thus we use the description of the molecule in a non-inertial reference frame, by means of a geometric approach of defects. Thus, in the neighborhood of the defects, a mixture of the K_{\pm} Fermi points occurs, and a generation of a non-Abelian gauge field, described for a t'Hooft-Polyakov monopole, takes place [44, 45]. So, we assume all 12 conical singularities of the molecule to be described by a Gödel-type metric. All this accords with the Osipov-Kolesnikov model [63], where eigenfunctions of the quasiparticles for small quantum levels do not oscillate too rapidly with distance. Also, we assume that our description works well in causal regions within the set of solutions $l^2 < 0$ thereby avoiding an imaginary vorticity $\Omega^2 < 0$.

Finally, our contribution naturally leads to additional terms in energy levels of the quasiparticles and current persists observed in the molecule that depends on rotation Ω . As well as, when compared with the different approaches about inertial frames [24, 63, 68, 69], the limits are well recovered. Also, in the slow rotation limit for the molecule ($\Omega \ll 1$), we obtain the results found in the references [26–28]. So, we conclude that the results found in this contribution, reveal the non-inertial effects in eigenvalues and eigenfunctions of energy, and persistent currents, for any general values of rotation about z-axis for the C_{60} fullerene.

Appendix: Gödel Solution

It is well known that Einstein's general relativity predicts several interesting phenomena, ranging from the classical time dilatation and contraction of space, to the curvature of space around heavy objects, such as the sun, a black hole, and most recently, the gravitational waves detected in recent months [72].

In the 80s, three works [35–37] examined in more detail the problem of causality in the Godel-type solutions in Einstein field equations. In all, it is possible to distinguish three different classes of solutions when we study the problem in cylindrical coordinates:

$$ds^2 = -[dt + H(r)d\phi]^2 + D^2(r)d\phi^2 + dr^2 + dz^2. \quad (\text{A.1})$$

Wherein $H(r) = \frac{\Omega}{l^2} \sinh^2(lr)$, and $D(r) = \frac{1}{2l} \sinh(2lr)$, where Ω and l are real constants. Also $\Omega = \frac{H'}{2D}$ and $l^2 = \frac{D''}{4D}$ are necessary conditions [38] for the homogeneity of space-time. Also [35–37] show that these conditions are not only necessary, but are sufficient for homogeneity since there are at least five linearly independent Killing vectors. The presence of CTCs is related to the behaviour of the function: $G(r) = D^2(r) - H^2(r)$. So, if $G(r)$ is negative in a given limited region, this region will have CTCs. We have three possibilities: (i) there are no CTCs, or $l^2 \geq \Omega^2$, (ii) there is an infinite sequence of alternating causal and non-causal regions, or $l^2 < 0$, and (iii) there is only one non-causal region, or $0 \leq l^2 < \Omega^2$.

One can also define three classes of solutions in metric as the symmetry of space-time with surfaces of constant curvature: (i) flat solutions, or rotation cosmic string, when $l^2 = 0$, (ii) solutions with positive spherical curvature when $l^2 < 0$, (iii) and hyperbolic solutions when $l^2 > 0$. Different aspects of the Gödel solutions are discussed also in [39–43].

Acknowledgements

We thanks to CAPES, CNPQ, CAPES/NANOBIOTEC and FAPESQ for financial support.

-
- [1] H. W. Kroto, J. R. Heath, S. C. O'Brien, R. F. Curl, R. E. Smalley, *Nature* **318**, 162 (1985).
 - [2] J. Gonzalez, F. Guinea, M. A. H. Vozmediano, *Phys. Rev. Lett.* **69**, 1 (1992).
 - [3] J. Gonzalez, F. Guinea, M. A. H. Vozmediano, *Nucl. Phys. B* **406**, 771 (1993).

- [4] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K.S. Novoselov, K. Geim, Rev. Mod. Phys. **81** (2009).
- [5] M. A. H. Vozmediano, M.I. Katsnelson, F. Guinea, Phys. Rep. **496**, 109 (2010).
- [6] J. K. Pachos, Contemp. Phys. **50**, 375 (2009).
- [7] M. Kleman, *Points, Lignes, Parois: dans les fluides anisotropes et les solides cristallins*, Editions de Physique, France, 1977.
- [8] M. O. Katanaev, I. V. Volovich, Ann. Phys. (N.Y.) **216**, 1 (1992).
- [9] C. Furtado, F. Moraes, A. M. de M. Carvalho, Phys. Lett. A **372**, 5368 (2008).
- [10] K. Bakke, C. Furtado, S. Sergeenkov, Europhys. Lett. **87**, 30002 (2009).
- [11] K. Bakke, A. Yu. Petrov and C. Furtado, Ann. of Phys. (N.Y.) **327**, 2946 (2012).
- [12] A. M. de M. Carvalho, C. A. de Lima Ribeiro, F. Moraes and C. Furtado, Eur. Phys. J. Plus **128**, 6 (2013).
- [13] M. J. Bueno, C. Furtado and A. M. de M. Carvalho, Eur. Phys. J. B, Cond. Mat. Phys. **85**, 53 (2012).
- [14] K. Bakke and C. Furtado, Phys. Rev. A **87**, 012130 (2013).
- [15] K. Bakke and C. Furtado, Phys. Lett. A **376**, 1269 (2012).
- [16] G. A. Marques, V. B. Bezerra, C. Furtado, F. Moraes, Int. J. Mod. Phys. A, **20**, 6051 (2005).
- [17] P. E. Lammert and V. H. Crespi, Phys. Rev. Lett. **85**, 5190 (2000).
- [18] P. E. Lammert and V. H. Crespi, Phys. Rev. B **69**, 035406 (2004).
- [19] N. D. Birrell, and P. C. W. Davies *Quantum fields in curved spaces* Cambridge (1982).
- [20] D. V. Kolesnikov and V. A. Osipov, Eur. Phys. J. B **49**, 465 (2006).
- [21] M. Pudlak, R. Pincak and V. A. Osipov, Phys. Rev. A **75**, 025201 (2007)
- [22] M. Pudlak, R. Pincak and V. A. Osipov, Phys. Rev. A **75**, 065201 (2007)
- [23] R. Pincak, Phys. Lett. A **340**, 267 (2005).
- [24] E. Cavalcante, C. Furtado, J. Phys. and Chem. of Solids **75** (2014) 1265-1268.
- [25] Y. Aharonov and G. Carmi, Found. Phys. **3**, ((1973) 493.
- [26] J. Q. Shen, S. He and F. Zhuang, Eur. Phys. J. D **33**, (2005) 35.
- [27] J. R. F. Lima, J. Brandão, M. M. Cunha, F. Moraes, Eur. Phys J. D **68** (2014) 94.
- [28] J. R. F. Lima and F. Moraes, Eur. Phys J. B **88** (2015) 263.
- [29] M. M. Cunha, J. Brandão, J. R. F. Lima and F. Moraes, Eur. Phys J. B **88** (2015) 288.
- [30] K. Imura, Y. Yoshimura, Y. Takane, T. Fukui, Phys. Rev. B **235119** (2012).

- [31] F. Mandl, G. Shaw, *Quantum Field Theory*, Second Edition, Wiley (2010).
- [32] K. Gödel, Rev Mod. Phys. **21** (1949) 447.
- [33] S. Hawking, Phys. Rev. D **46** (1992) 603.
- [34] H. Monroe, Found. of Phys. **38** (11) (2008) 1065.
- [35] M. Rebouças, J. Tiomno, Phys. Rev. D **28**(1983) 1251.
- [36] M. Rebouças, M. Aman, A. F. F. Teixeira, J. Math. Phys. **27** (1985) 1370.
- [37] M. O. Galvão, M. Rebouças, A. F. F. Teixeira, W. M. Silva Jr, Math. Phys. **29** (1988) 1127.
- [38] A. K. Raychaudhuri, S. N. Thakurta, Phys. Rev. D **22** (1980) 802.
- [39] J. D. Barrow, C. Tsagas, Class. Quantum Gravity **21** (2004) 1773.
- [40] J. D. Barrow, C. Tsagas, Phys. Rev. D **69** (2004) 064007.
- [41] T. Clifton, J. Barrow, Phys. Rev. D **72**(2005) 123003.
- [42] R. J. Gleiser, M. Gurses, A. Karasu, S. Özgür, Class. Quantum Gravity **23** (2006) 2653.
- [43] J. Carvalho, A. M. de M. Carvalho, C Furtado, Eur. Phys. J. C **74** (2014) 2935.
- [44] J. Gonzalez, F. Guinea, M. A. H. Vozmediano, Phys. Rev. Lett. **69** (1992).
- [45] J. Gonzalez, F. Guinea, M. A. H. Vozmediano, Nucl. Phys. B **406** (1993) 771.
- [46] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, K. Geim, Rev. Mod. Phys. **81** (2009).
- [47] T. Vachaspati, Phys. Rev. D **44** (1991) 3723.
- [48] T. W. B. Kibble, J. Phys. A **9** (1976) 1387.
- [49] A. Vilenkin, Phys. Rev. D **23** (1981) 852.
- [50] B. Linet, Phys. Rev. D **33** (1986) 1333.
- [51] M. Barriola, A. Vilenkin, Phys. Rev. Lett. **63**(1976) 341.
- [52] F. C. Frank, *On the theory of liquid crystals*, Discussion Farad. Soc. **25** (1958) 19-28.
- [53] I. E. Dzyaloshinskii, G. E. Volovich, Physique **39(6)** (1978) 693-700.
- [54] J. A. Hertz, Phys. Rev. B **18** (1978) 4875-4885.
- [55] C. Furtado, F. Moraes, A. M. de M. Carvalho, Phys. Lett. A **372** (2008) 5368.
- [56] A. M. de M. Carvalho, C. A. de Lima Ribeiro, F. Moraes, C. Furtado, Eur. Phys. J Plus **128** (2013) 6.
- [57] M. J. Bueno, C. Furtado, A. M. de M. Carvalho, Eur. Phys. J. B, Cond. Mat. Phys **85** (2012) 53.
- [58] K. Bakke, C. Furtado, Phys. Rev. A **87** (2013) 012130.

- [59] G. A. Marques, V. B. Bezerra, C. Furtado, F. Moraes, Int. J. Mod. Phys. A **20** (2005) 6051.
- [60] M. A. H. Vozmediano, M. I. Katsnelson, F. Guinea, Phys. Rep. **496** (2010) 109.
- [61] P. E. Lammert, V. H. Crespi, Phys. Rev. Lett. **85** (2000) 5196.
- [62] P. E. Lammert, V. H. Crespi, Phys. Rev. B **69** (2004) 035406.
- [63] D. V. Kolesnikov, V. A. Osipov, Eur. Phys. J. B **49** (2006) 465.
- [64] N. D. Birrel, P. C. W. Davies, *Quantum Fields in Curved Spaces*, Cambridge (1982).
- [65] S Coleman, *The Magnetic Monopole Fifty Years Later, in The Unity of Fundamental Interactions*, Plenum, New York (1983).
- [66] C. Furtado, V. B. Bezerra, F. Moraes, Phys. Lett. A **289** (2001) 160-166.
- [67] J. S. Carvalho, E. Passos, C. Furtado, F. Moraes, Eur. Phys. J. C **57** (2008) 817.
- [68] K. Imura, Y. Yoshimura, Y. Takane, T. Fukui, Phys. Rev. B (2012) 235119.
- [69] A. A. Abrikosov, Jr., hep-th/0212134.
- [70] M. Buttiker, Y. Imry, R. Landauer, Phys. Lett. A **96(7)** (1983) 365.
- [71] N. Byers, C. N. Yang, Phys. Rev. Lett. **7** (1961) 46.
- [72] B. P. Abbott, et. al., Phys. Rev. Lett. **116** (2016) 061102.